

B.21

1891371

$$a) f(x) = \frac{x^4 - 4x^2}{(x-4)^2(x-1)^3}; \quad f'(x) = \frac{(x^4 - 4x^2)'(x-4)^2(x-1)^3 - (x^4 - 4x^2)[(x-4)^2'(x-1)^3 + (x-4)^2(x-1)^3']}{[(x-4)^2(x-1)^3]^2} =$$

$$= \frac{(4x^3 - 8x)(x-4)^2(x-1)^3 - (x^4 - 4x^2)[2(x-4)(x-1)^3 + 3(x-1)^2(x-4)^2]}{(x-4)^4(x-1)^6}$$

$$b) f(x) = \frac{(4x-1)(x+1)^2}{(2x+1)(x-1)^2}$$

$$f'(x) = \frac{[(4x-1)(x+1)^2]'(2x+1)(x-1)^2 - (4x-1)(x+1)^2[(2x+1)(x-1)^2]'}{[(2x+1)(x-1)^2]^2} =$$

$$= \frac{[4(x+1)^2 + (4x-1)2(x+1)](2x+1)(x-1)^2 - (4x-1)(x+1)^2[2(x-1)^2 + (2x+1)2(x-1)]}{(2x+1)^2(x-1)^4}$$

$$b) f(x) = \frac{(x-1)(x+2)^2}{(2x-1)^2}$$

$$f'(x) = \frac{[(x-1)(x+2)^2]'(2x-1)^2 - (x-1)(x+2)^2[(2x-1)^2]'}{(2x-1)^4} =$$

$$= \frac{[(x+2)^2 + 2(x+2)(x-1)](2x-1)^2 - (x-1)(x+2)^2 \cdot 2(2x-1) \cdot 2}{(2x-1)^4}$$

$$z) f(x) = \frac{1}{1+x} \quad f'(x) = -\frac{1}{(1+x)^2}$$

B.22

$$a) f(x) = \frac{(x-4)(x^2-9)}{(x^2-5x+6)}$$

$$f'(x) = \frac{[(x-4)(x^2-9)]'(x^2-5x+6) - (x^2-5x+6)'(x-4)(x^2-9)}{(x^2-5x+6)^2}$$

$$= \frac{[(x^2-9) + 2x(x-4)](x^2-5x+6) - (2x-5)(x-4)(x^2-9)}{(x^2-5x+6)^2}$$

$$b) f(x) = x \frac{x}{\sqrt{4-x}} = \frac{x^2}{\sqrt{4-x}}$$

$$f'(x) = \frac{(x^2)'\sqrt{4-x} - x^2(\sqrt{4-x})'}{4x(4-x) + x^2 4 - x} = \frac{2x\sqrt{4-x} + x^2\left(+\frac{1}{2\sqrt{4-x}}\right)'}{4-x} =$$

$$= \frac{2(4-x)\sqrt{4-x}}{2(4-x)\sqrt{4-x}}$$

$$b) f(x) = \frac{x^4 - 4x^3 + 2x^2}{(x-2)(x-1)}$$

~~$$f'(x) = \frac{(x^4 - 4x^3 + 2x^2)'(x-2)(x-1) - (x^4 - 4x^3 + 2x^2)[(x-2)(x-1)]'}{(x-2)^2(x-1)^2}$$~~

$$f'(x) = \frac{(x^4 - 4x^3 + 2x^2)'(x-2)(x-1) - (x^4 - 4x^3 + 2x^2)[(x-2)(x-1)]'}{(x-2)^2(x-1)^2} =$$

$$= \frac{(4x^3 - 12x^2 + 4x)(x-2)(x-1) - (x^4 - 4x^3 + 2x^2)[(x-1) + (x+2)]}{(x-2)^2(x-1)^2} =$$

$$= \frac{(4x^3 - 12x^2 + 4x)(x-2)(x-1) - (x^4 - 4x^3 + 2x^2)(2x+1)}{(x-2)^2(x-1)^2}$$

$$c) f(x) = \sqrt[3]{x^2 - x^3} + \frac{1}{\sqrt{x+2}}$$

$$f'(x) = \frac{1}{3}(x^2 - x^3)^{-\frac{2}{3}}(2x - 3x^2) - \frac{1}{2(x+2)\sqrt{x+2}} =$$

$$= \frac{2x - 3x^2}{3\sqrt[3]{(x^2 - x^3)^2}} - \frac{1}{2(x+2)\sqrt{x+2}}$$

$$2) f(x) = \frac{x^3 + 4x^2 + 4x}{x^2 - 9}$$

$$f'(x) = \frac{(x^3 + 4x^2 + 4x)'(x^2 - 9) - (x^3 + 4x^2 + 4x)(x^2 - 9)'}{(x^2 - 9)^2} =$$

$$= \frac{(3x^2 + 8x + 4)(x^2 - 9) - (x^3 + 4x^2 + 4x) \cdot 2x}{(x^2 - 9)^2}$$

$$e) f(x) = \sqrt[3]{x^3 - 3x^2} = (x^3 - 3x^2)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}(x^3 - 3x^2)^{-\frac{2}{3}}(3x^2 - 6x) = \frac{3x^2 - 6x}{3\sqrt[3]{(x^3 - 3x^2)^2}}$$

$$m) f(x) = \frac{x^3}{1 - x^2}$$

$$f'(x) = \frac{(x^3)'(1 - x^2) - x^3(1 - x^2)'}{(1 - x^2)^2} = \frac{3x^2(1 - x^2) + x^3 \cdot 2x}{(1 - x^2)^2}$$

$$= \frac{3x^2(1 - x^2) + 2x^4}{(1 - x^2)^2}$$

$$\text{B.23} \quad a) f(x) = x + \frac{1}{x^2} = x + x^{-2}$$

$$f'(x) = 1 - 2x^{-3} = 1 - \frac{2}{x^3}$$

$$8) f(x) = \sqrt{(x-1)^2(x-2)}$$

$$f'(x) = \frac{2(x-1)(x-2) + (x-1)^2}{2\sqrt{(x-1)^2(x-2)}}$$

$$b) f(x) = \frac{x^4 + 2x^3 - 3x^2}{(x^2 - 4)(x+1)^2}$$

$$f'(x) = \frac{(x^4 + 2x^3 - 3x^2)'(x^2 - 4)(x+1)^2 - (x^4 + 2x^3 - 3x^2)[(x^2 - 4)(x+1)^2]'}{(x^2 - 4)^2(x+1)^4} =$$

$$= \frac{(4x^3 + 6x^2 - 6x)(x^2 - 4)(x+1)^2 - (x^4 + 2x^3 - 3x^2) [2x(x+1)^2 + (x^2 - 4) \cdot 2(x+1)]}{(x^2 - 4)^2 (x+1)^4}$$

$$2) f(x) = \frac{(x+2)\sqrt{x}}{x-1}$$

$$f'(x) = \frac{[(x+2)\sqrt{x}]'(x-1) - (x+2)\sqrt{x}(x-1)'}{(x-1)^2} =$$

$$= \frac{[\sqrt{x} + \frac{x+2}{2\sqrt{x}}](x-1) - (x+2)\sqrt{x}}{(x-1)^2}$$

$$3) f(x) = \frac{((x+1)(x-2))^2}{(x-3)(x^2+1)} = \frac{(x+1)^2(x-2)^2}{(x-3)(x^2+1)}$$

$$f'(x) = \frac{[(x+1)^2(x-2)^2]'(x-3)(x^2+1) - (x+1)^2(x-2)^2[(x-3)(x^2+1)]'}{(x-3)^2(x^2+1)^2}$$

$$= \frac{[2(x+1)(x-2)^2 + (x+1)^2 2(x-2)](x-3)(x^2+1) - (x+1)^2(x-2)^2[x^2-1+2x(x-3)]}{(x-3)^2(x^2+1)^2}$$

$$e) f(x) = \frac{\sqrt[3]{x^2-x}}{x^2-x-6} \quad f'(x) = \frac{[(x^2-x)^{\frac{1}{3}}]'(x^2-x-6) - \sqrt[3]{x^2-x}(x^2-x-6)'}{(x^2-x-6)^2}$$

$$= \frac{\frac{2x-1}{3\sqrt[3]{(x^2-x)^2}}(x^2-x-6) - \sqrt[3]{x^2-x}(2x-1)}{(x^2-x-6)^2}$$

$$= \frac{(2x-1)(x^2-x-6) - 3(x^2-x)(2x-1)}{3\sqrt[3]{(x^2-x)^2}(x^2-x-6)^2}$$

$$м) f(x) = \frac{1}{\cos x} \quad f'(x) = \frac{\sin x}{\cos^2 x}$$

Извини за гръб. Спешно.