① $S(t) = 3t^3 - 2t^2 + 4t$ (м)

$v(t) = S'(t) = (3t^2 - 2t^2 + 4t)' =$

$= 9t^2 - 4t + 4$

$v(1) = 9 \cdot 1 - 4 \cdot 1 + 4 = 9$ м/с

Отв: 9 м/с

② $\vec{a} = (n; 6; -3)$; $\vec{b} = (-2; m; 1)$

$\frac{n}{-2} = \frac{6}{m} = \frac{-3}{1} = -3$

$\frac{n}{-2} = -3 \Rightarrow n = 6$

$\frac{6}{m} = -3 \Rightarrow -3m = 6 \Rightarrow m = -2$

Отв: $n = 6$, $m = -2$

③ $\int (7x^2 + x^3)\,dx = 7\int x^2\,dx + \int x^3\,dx =$

$= \frac{7x^3}{3} + \frac{x^4}{4} + C$

④ $f(x) = (x^2 - 2) \cdot \sin x$

$f'(x) = (x^2 - 2)' \sin x + (x^2 - 2)(\sin x)' =$

$= 2x \cdot \sin x + (x^2 - 2)\cos x$

⑤ $A(3;1)$; $B(4;2)$.

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

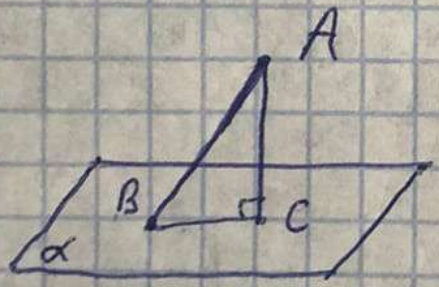
$$\frac{x-3}{4-3} = \frac{y-1}{2-1}$$

$$x-3 = y-1$$

$$y = x-2$$

Отв: $y = x-2$

⑥



$AC = 12$;

$AB = 13$

$BC - ?$

$$BC = \sqrt{AB^2 - AC^2} = \sqrt{169 - 144} =$$

$$= \sqrt{25} = 5 \text{ см}$$

Отв: 5 см

⑦ $f(x) = -3\operatorname{ctg} 4x, \quad x_0 = \frac{\pi}{8}$

$f'(x) = (-3\operatorname{ctg} 4x)' \cdot (4x)' = -3 \cdot \left(-\frac{1}{\sin^2 4x}\right) \cdot 4 =$

$= \frac{12}{\sin^2(4x)}$

$f'(x_0) = \frac{12}{\sin^2\left(4 \cdot \frac{\pi}{8}\right)} = \frac{12}{\sin^2\left(\frac{\pi}{2}\right)} = \frac{12}{1} = \boxed{12}$

⑧ $\lim\limits_{x \to \infty} \frac{5x^4 - x^3 + 2x}{x^4 - 3x + 1} = \lim\limits_{x \to \infty} \frac{\frac{5x^4}{x^4} - \left(\frac{x^3}{x^4}\right)^{\to 0} + \left(\frac{2x}{x^4}\right)^{\to 0}}{\frac{x^4}{x^4} - \left(\frac{3x}{x^4}\right)_{\to 0} + \left(\frac{1}{x^4}\right)_{\to 0}} =$

$= \frac{5}{1} = \boxed{5}$

⑨ $f(x) = 2x^3 - 3x, \quad x_0 = 3$

$y_k(x) = f(x_0) + f'(x_0)(x - x_0)$

$f(x_0) = 3^3 \cdot 2 - 3 \cdot 3 = 54 - 9 = 45$

$f'(x) = (2x^3 - 3x)' = 6x^2 - 3$

$f'(x_0) = 6 \cdot 3^2 - 3 = 54 - 3 = 51$

$y_k(x) = 45 + 51(x - 3) = 45 + 51x - 153 =$

$= 51x - 108$

Отв: $y_k(x) = 51x - 108$

(10) $\bar{m} = 3\bar{a} - \bar{b}$

$\bar{a} = (-3; 2; 0)$; $\bar{b} = (1; -3; 2)$

$\bar{m} = 3\bar{a} - \bar{b}$

$3\bar{a} = (-3 \cdot 3;\ 2 \cdot 3;\ 0 \cdot 3) = (-9; 6; 0)$

$\bar{m} = (-9 - 1;\ 6 + 3;\ 0 - 2) =$

$= (-10; 9; -2)$

Отв: $\underline{\bar{m} = \{-10; 9; -2\}}$

(11) $\int \cos^3 x \sin x\, dx = \int \cos^3 x \cdot \sin x \cdot \left(-\frac{1}{\sin x}\right) d(\cos x)$

$= -\int \cos^3 x\, d(\cos x) = \underline{-\frac{\cos^4 x}{4} + C}$

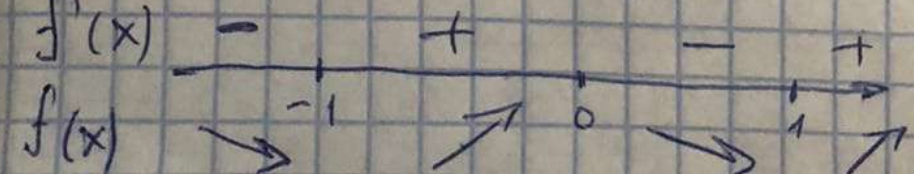
(12) $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + \frac{1}{8}$

$f'(x) = x^3 - x$

Точки экстремума: $f'(x) = 0 \Rightarrow x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0, \quad x = 1; \quad x = -1$



$f(-1) = \frac{1}{4}(-1)^4 - \frac{1}{2}(-1)^2 + \frac{1}{8} = \frac{1}{4} - \frac{1}{2} + \frac{1}{8} =$

$= \frac{1}{4} - \frac{2}{4} + \frac{1}{8} = -\frac{2}{8} + \frac{1}{8} = -\frac{1}{8}$ - min

$f(0) = \frac{1}{8}$ - max

$f(1) = -\frac{1}{8}$ - min

$x = 1; -1$ - точка min функции

$x = 0$ - точка max функции

$(-\infty; -1) \cup (0; 1)$ - функция убывает

$(-1; 0) \cup (1; +\infty)$ - функция возрастает

(13) $\lim\limits_{x \to 1} \frac{x-1}{x^2-10x+9}$

$x^2 - 10x + 9 = 0$; $D = 100 - 36 = 64$; $\sqrt{D} = 8$

$x_1 = \frac{10+8}{2} = 9$; $x_2 = \frac{10-8}{2} = 1$

$\lim\limits_{x \to 1} \frac{x-1}{x^2-10x+9} = \lim\limits_{x \to 1} \frac{x-1}{(x-9)(x-1)} = \lim\limits_{x \to 1} \frac{1}{x-9} =$

$= \frac{1}{1-9} = -\frac{1}{8}$

(14) $v_1 = t + 1$ м/с

$v_2 = t^3 - 3t$ м/с

$t = 6$ с

$v(t) = S'(t) \Rightarrow S(t) = \int_0^t v(t)\,dt$

$S_1(t) = \int_0^6 (t+1)\,dt = \frac{t^2}{2} + t \Big|_0^6$

$S_2(t) = \int_0^6 (t^3 - 3t)\,dt = \frac{t^4}{4} - \frac{3t^2}{2} \Big|_0^6$

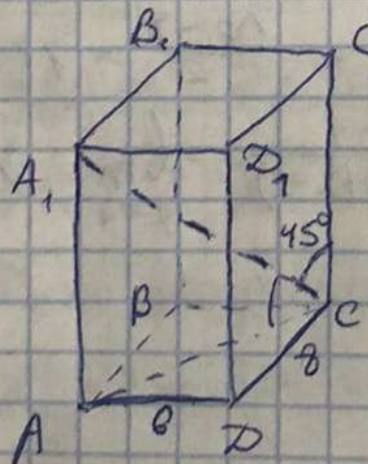
$S_1(6) = \frac{6^2}{2} + 6 = 18 + 6 = 24$ м

$S_2(6) = \frac{6^4}{4} - \frac{3 \cdot 6^2}{2} = 324 - 54 = 270$ м

$270 - 24 = 246$ м

Отв: 246 м

(15)



$AD = 6$; $DC = 8$

A_1C – диагональ

$\angle A_1CC_1 = 45°$

$V - ?$

$\measuredangle AA_1C_1C$:

$\angle A_1CC_1 = 45° \Rightarrow \angle A_1CA = 45°$

$\angle CA_1A = \angle A_1CC_1 = 45° \Rightarrow A_1A = AC$

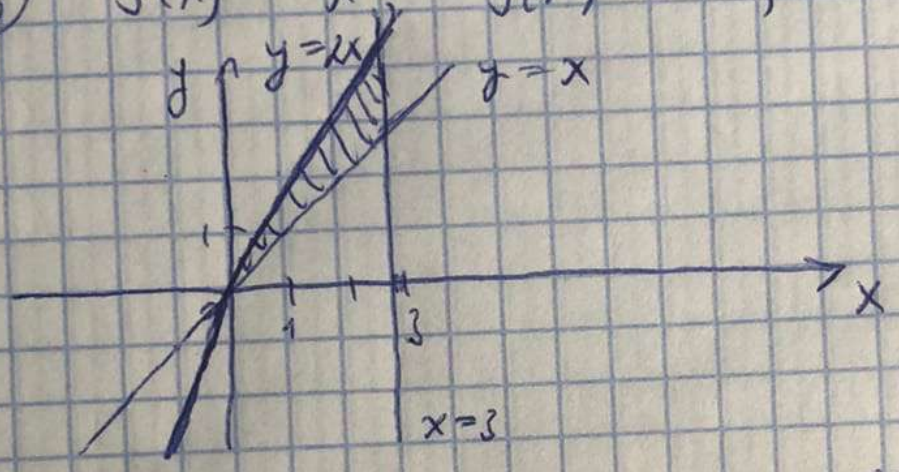
$AC = \sqrt{AD^2 + DC^2} = \sqrt{36 + 64} = 10$ см $\Rightarrow$

$\Rightarrow AA_1 = 10$ см

$V = AA_1 \cdot AD \cdot DC = 10 \cdot 6 \cdot 8 = 480$ см^3

Ответ: $\underline{V = 480 \text{ см}^3}$

(16) $f(x) = x$, $f(x) = 2x$, $x = 3$



$$S = \int_0^3 2x\,dx - \int_0^3 x\,dx = \int_0^3 x\,dx =$$

$$= \frac{x^2}{2}\Big|_0^3 = \frac{9}{2} = 4{,}5 \text{ (кв. ед.)}$$

Ответ: 4,5 (кв. ед.)