task/1843193

14) $y=\frac{x^2-x+1}{x^2+x+1}$

$$y'=\frac{(x^2-x+1)'(x^2+x+1)-(x^2-x+1)(x^2+x+1)'}{(x^2+x+1)^2}=$$

$$=\frac{(2x-1)(x^2+x+1)-(x^2-x+1)(2x+1)}{(x^2+x+1)^2}$$

15) $y=\frac{\lg x}{\sqrt{x}}$ $\quad y'=\frac{(\lg x)'\sqrt{x}-\lg x(\sqrt{x})'}{(\sqrt{x})^2}=$

$$=\frac{\frac{1}{x\ln 10}\cdot\sqrt{x}-\lg x(-\frac{1}{2}x^{-\frac{1}{2}})}{x}=\frac{\sqrt{x}}{x^2\ln 10}+\frac{\lg x}{2x\sqrt{x}}$$

16) $y=\frac{2x}{\sin x+1}$ $\quad y'=\frac{(2x)'(\sin x+1)-2x(\sin x+1)'}{(\sin x+1)^2}=$

$$=\frac{2(\sin x+1)-2x\cos x}{(\sin x+1)^2}=\frac{2}{\sin x+1}-\frac{2x\cos x}{(\sin x+1)^2}$$

17) $y=\sin^3 x-\sqrt{3}\cos^3 x \quad x_0=60^\circ$

$$y'=3\sin x\cdot\cos x+\sqrt{3}\cdot 3\cos x\cdot\sin x=3\sin x\cos x(1+\sqrt{3})$$

$$y'|_{x_0=60^\circ}=3\sin 60\cdot\cos 60(1+\sqrt{3})=3\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}(1+\sqrt{3})=$$

$$=\frac{3\sqrt{3}+3\sqrt{3}\sqrt{3}}{4}=\frac{3\sqrt{3}+9}{4}=\frac{3}{4}(\sqrt{3}+3)$$

18) $y=e^{2x}-e^{-3x}-x \quad x_0=0$

$$y'=e^{2x}\cdot 2+e^{-3x}\cdot 3-1$$

$$y'|_{x=0}=2e^{2\cdot 0}+3e^{-3\cdot 0}-1=2+3-1=4$$

19) $y=\cos(6x+\frac{\pi}{2})-\sin(\pi-6x) \quad x_0=\frac{\pi}{6}$

$$y'=-6\sin(6x+\frac{\pi}{2})+6\cos(\pi-6x)$$

$$y'|_{x=\frac{\pi}{6}}=-6\sin(6\frac{\pi}{6}+\frac{\pi}{2})+6\cos(\pi-6\frac{\pi}{6})=$$

$$=-6\sin\frac{3\pi}{2}+6\cos 0=6+6=12$$

20) $y = \dfrac{\cos x}{1+\sin x}$ $\quad x_0 = \dfrac{\pi}{2}$

$$y' = \frac{(\cos x)'(1+\sin x) - \cos x\,(1+\sin x)'}{(1+\sin x)^2} =$$

$$= \frac{-\sin x(1+\sin x) - \cos x \cdot \cos x}{(1+\sin x)^2} = -\frac{\sin x + \sin^2 x + \cos^2 x}{(1+\sin x)^2} =$$

$$= -\frac{1+\sin x}{(1+\sin x)^2} = -\frac{1}{1+\sin x}$$

$$y'\Big|_{x=\frac{\pi}{2}} = -\frac{1}{1+\sin\frac{\pi}{2}} = -\frac{1}{1+1} = -\frac{1}{2}$$