$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin x \cdot \cos 5x\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{2}\left(\sin 6x - \sin 4x\right)dx =$$

$$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin 6x\,dx - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin 4x\,dx = \frac{1}{12}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin 6x\,d6x - \frac{1}{8}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin 4x\,d4x =$$

$$= -\frac{1}{12}\cos 6x + \frac{1}{8}\cos 4x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = -\frac{1}{12}\cos\frac{6\pi}{4} + \frac{1}{8}\cos\frac{4\pi}{4} +$$

$$+ \frac{1}{12}\cos\frac{6\pi}{6} - \frac{1}{8}\cos\frac{4\pi}{6} = -\frac{1}{12}\cos\frac{3\pi}{2} + \frac{1}{8}\cos\pi + \frac{1}{12}\cos\pi - \frac{1}{8}\cos\frac{2\pi}{3}$$

$$= 0 - \frac{1}{8} - \frac{1}{12} - \frac{1}{8}\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = -\frac{1}{8} - \frac{1}{12} + \frac{1}{8}\sin\frac{\pi}{6} =$$

$$= -\frac{6/}{1}\frac{}{8} - \frac{4/}{1}\frac{}{12} + \frac{3/}{1}\frac{}{16} = -\frac{6+4-3}{48} = -\frac{7}{48}$$