4351031

1) а) $\log_3 81 - \ln e + \lg 1000 =$

$= \log_3 3^4 - \ln e + \lg 10^3 = 4 - 1 + 3 = 6$

б) $\dfrac{2 \cdot \log_7 16}{\left(\log_3(\sqrt{10}+1) + \log_3(\sqrt{10}-1)\right)\log_7 2} =$

$= \dfrac{\log_7 16^2}{\log_7 2} \cdot \dfrac{1}{\log_3\left((\sqrt{10}+1)\cdot(\sqrt{10}-1)\right)} = \dfrac{\log_2 2^5}{\log_3(10-1)} =$

$= \dfrac{5}{\log_3 3^2} = \dfrac{5}{2} = 2{,}5$

2. а) $4^x - 3\cdot 2^x + 2 = 0 \Rightarrow 2^{2x} - 3\cdot 2^x + 2 = 0 \Rightarrow 2^x = y$

$y^2 - 3y + 2 = 0 \Rightarrow y_1 = 2 \quad y_2 = 1 \Rightarrow 2^{x_1} = 2 \Rightarrow$

$x_1 = 1 \; ; \; 2^{x_2} = 1 \Rightarrow x_2 = 0$

б) $\log_2 x + 6\log_4 x = 8 \Rightarrow \log_2 x + 6\log_{2^2} x = 8 \Rightarrow$

$\Rightarrow \log_2 x + \dfrac{6}{2}\log_2 x = 8 \Rightarrow 4\log_2 x = 8 \Rightarrow \log_2 x = 2 \Rightarrow$

$\Rightarrow x = 2^2 = 4$

3. а) $3^{x+2} - 2\cdot 3^{x+1} + 3^x < 12 \Rightarrow 9\cdot 3^x - 6\cdot 3^x + 3x < 12 \Rightarrow$

$\Rightarrow 4\cdot 3^x < 12 \Rightarrow 3^x < 3 \quad x < 1$

б) $(\log_{0,5} x)^2 + 3\log_{0,5} x - 4 \le 0 \qquad ОДЗ: x > 0$

$(\log_{0,5} x + 4)(\log_{0,5} x - 1) \le 0$

$-4 \le \log_{0,5} x \le 1 \qquad \log_{0,5} x = \log_{2^{-1}} x = -\log_2 x$

$\log_2 x \le -4 \qquad \log_2 x \ge 1$

$x \le 2^{-4} \qquad x \ge 2$

$x \le \frac{1}{16}$

$(0; \frac{1}{16}] \cup [2; \infty)$

4. $(\sqrt{5})^{\log_5(\sqrt{2}-1)^2} + (\sqrt{3})^{\log_3(\sqrt{2}-2)^2} = 1$

$5^{\frac{1}{2}\cdot\log_5(\sqrt{2}-1)^2} + 3^{\frac{1}{2}\log(\sqrt{2}-2)^2} = 5^{\log_5(\sqrt{2}-1)} + 3^{\log_3(\sqrt{2}-2)} =$

$= \sqrt{2}-1+\sqrt{2}-2 = 2\sqrt{2}-3 \neq 1$

$7^{\log_{27}8} : 2^{\log_3 7} = 7^{\frac{3}{3}\log_3 2} : 2^{\log_3 7} = 2^{\log_3 7} : 2^{\log_3 7} =$

$= 1$

$5\cdot\left(\frac{5}{6}\right)^{x-1} - 9\left(\frac{6}{5}\right)^x + 3 = 0$

$5\cdot\frac{6}{5}\cdot\left(\frac{5}{6}\right)^x - 9\left(\frac{6}{5}\right)^x + 3 = 0$

$6\cdot\left(\frac{5}{6}\right)^x - \frac{9}{\left(\frac{5}{6}\right)^x} + 3 = 0$

$6\left(\frac{5}{6}\right)^{2x} + 3\left(\frac{5}{6}\right)^x - 9 = 0 \qquad \left(\frac{5}{6}\right)^x = y$

$2y^2 + y - 3 = 0 \qquad y_{1,2} = \frac{-1 \pm \sqrt{1+24}}{4} = \frac{-1 \pm 5}{4}$

$y_1 = -\frac{6}{5}$ не подходит, т.к $\left(\frac{5}{6}\right)^x$ всегда > 0

$y_2 = 1$

$\left(\frac{5}{6}\right)^x = 1 \quad x = 0$

Всего доброго!