

Решить систему уравнений

$$\begin{cases} x^2 + xy + y^2 = 7 & * (x - y) \\ y^2 + yz + z^2 = 3 & * (y - z) \\ z^2 + xz + x^2 = 1 & * (z - x) \end{cases}$$

$$\begin{cases} x^3 - y^3 = 7(x - y) \\ y^3 - z^3 = 3(y - z) \\ z^3 - x^3 = (z - x) \end{cases}$$

$$0 = 7x - 7y + 3y - 3z + z - x; \quad z = 3x - 2y$$

$$\begin{cases} y^2 + yz + z^2 = 3 \\ z^2 + xz + x^2 = 1 \end{cases}$$

$$y^2 + yz - xz - x^2 = 2;$$

$$y^2 + z(y - x) - x^2 = 2;$$

$$y^2 + (y - x)(3x - 2y) - x^2 = 2;$$

$$y^2 + 3xy - 3x^2 - 2y^2 + 2xy - x^2 = 2$$

$$\begin{cases} -y^2 + 5xy - 4x^2 = 2 & * (-7) \\ y^2 + xy + x^2 = 7 & * 2 \end{cases}$$

$$\begin{cases} 7y^2 - 35xy + 28x^2 = -14 \\ 2y^2 + 2xy + 2x^2 = 14 \end{cases}$$

$$9y^2 - 33xy + 30x^2 = 0; \quad 3y^2 - 11xy + 10x^2 = 0;$$

$$D = 121x^2 - 4 * 3 * 10x^2 = x^2$$

$$1)y = \frac{11x + x}{6} = 2x; \text{ подставим в 1 уравнение}$$

$$x^2 + 2x^2 + 4x^2 = 7; x^2 = 1$$

$$x_1 = 1; y_1 = 2; z_1 = 3 * 1 - 2 * 2 = -1$$

$$x_2 = -1; y_2 = -2; z_2 = 3 * (-1) - 2 * (-2) = 1$$

$$2)y = \frac{11x - x}{6} = \frac{5}{3}x; \text{ подставим в 1 уравнение}$$

$$x^2 + \frac{5x^2}{3} + \frac{25x^2}{9} = 7; \quad 9x^2 + 15x^2 + 25x^2 = 63;$$

$$49x^2 = 63; \quad 7x^2 = 9; \quad x^2 = \frac{9}{7}$$

$$x_3 = -\frac{3}{\sqrt{7}}; \quad y_3 = -\frac{5}{\sqrt{7}}; \quad z_3 = -\frac{9}{\sqrt{7}} - 2 * \left(-\frac{5}{\sqrt{7}}\right) = \frac{1}{\sqrt{7}}$$

$$x_4 = \frac{3}{\sqrt{7}}; \quad y_4 = \frac{5}{\sqrt{7}}; \quad z_4 = \frac{9}{\sqrt{7}} - 2 * \frac{5}{\sqrt{7}} = -\frac{1}{\sqrt{7}}$$

Ответ: $(-1; -2; 1)(1; 2; -1)$

$$\left(-\frac{3}{\sqrt{7}}; -\frac{5}{\sqrt{7}}; \frac{1}{\sqrt{7}}\right) \left(\frac{3}{\sqrt{7}}; \frac{5}{\sqrt{7}}; -\frac{1}{\sqrt{7}}\right)$$